An Anisotropic Micromechanical Model for Granular Materials: Development and Validation of a Novel Contact Law

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Abstract. The consideration of fabric anisotropy in the simulation of granular materials is crucial in geotechnical engineering. Conventional models for granular materials that account for fabric anisotropy are typically based on the anisotropic critical state theory, where a fabric tensor is incorporated in the critical state formulation. However, the formulation of the fabric tensor in critical state theory is largely empirical and lacks a clear physical interpretation. A promising alternative is the multi-scale micromechanical model, which directly incorporates fabric anisotropy through the averaging of microscale quantities. The CH micromechanical model is an inter-particle-plane-based framework in which inelastic behavior is fully defined at inter-particle contacts. Although the CH micromechanical model has been extended to simulate unsaturated soils, clays, grouted soils and lunar soils by modifying the microscale Coulomb-type contact law, the potential of integrating arbitrary contact laws into the CH micromechanical framework has not been fully explored. In this paper, we introduced a novel contact law into the CH micromechanical model to simulate granular materials with fabric anisotropy. The performance of the model is validated by comparing simulation results with experimental data from triaxial tests on sand. These results demonstrate that the integrated model effectively captures fabric anisotropy in granular material simulations, offering a promising alternative for geotechnical applications.

Keywords: Fabric Anisotropy · Granular Materials · Multiscale · Micromechanical modelling

1 Introduction

Granular materials consist of clusters of particles that interact through contact forces. The mechanical behavior of granular materials is complex due to their discontinuous nature and the presence of multiple scales of heterogeneity. Researchers introduced the fabric tensor into the critical state line (Li and Dafalias, 2002; Wan and Guo, 2001), dilatancy

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state line and plastic modulus (Li and Dafalias, 2012; Papadimitriou et al., 2019; Petalas et al., 2019), yield surface and potential function (Gao et al., 2014; Gao et al., 2024; Gao and Zhao, 2017) of existing macroscopic models to account for the anisotropy of granular materials. Although these models incorporate the fabric anisotropy with the introduction of fabric tensors, they are fundamentally phenomenological models and lack clear physical meanings.

The CH micromechanical model (Chang and Hicher, 2005) is a promising alternative that directly incorporates fabric anisotropy through the averaging of microscale quantities without relying empirical assumptions on the fabric tensor. Although researchers have extended the CH micromechanical model to simulate unsaturated soils (Hicher and Chang, 2007; Zhao et al., 2017), clays (Chang et al., 2009; Yin et al., 2014), grouted soils (Hicher et al., 2008), and lunar soils (Chang and Hicher, 2009) by modifying the original Coulomb-type contact law, the potential of integrating arbitrary contact laws into the CH micromechanical framework has not been fully explored. Therefore, in this paper, we propose a novel contact law into the CH micromechanical model to simulate granular materials with fabric anisotropy. The performance of the model is validated by comparing simulation results with experimental data from triaxial tests on sand.

2 Introduction of the CH Micromechanical Model

2.1 Basic Framework

The CH micromechanical model was initially proposed by Chang and Hicher (2005) for sand. It was then extended to simulate unsaturated soils (Hicher and Chang, 2007; Zhao et al., 2017) and clay (Chang et al., 2009; Yin et al., 2014). The framework of the CH micromechanical model shown in Fig. 1 consists of 4 steps: (1) localization; (2) contact law integration; (3) averaging; (4) unbalanced force checking. Detailed introduction of the CH micromechanical model and its implementation can be found in Chang and Hicher (2005) and Zhao et al. (2018).

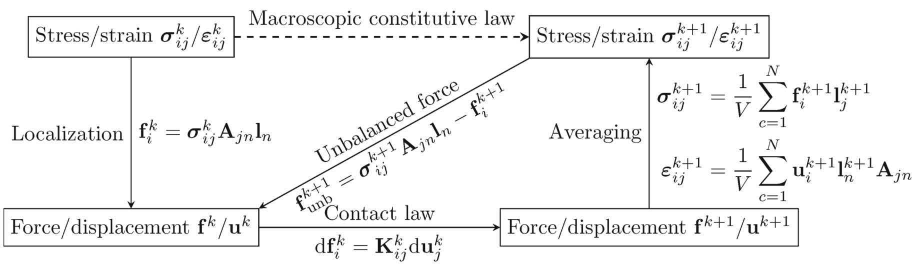

Fig. 1. Framework of the CH micromechanical model

2.2 Fabric Anisotropy

One of the most important features of the multi-scale micromechanical model is that it can simulate fabric anisotropy of granular materials through the integration of contact forces. The fabric anisotropy can be characterized by the fabric tensor, which can be defined as (Chang and Misra, 1990):

$$\mathbf{F}_{ij} = \begin{bmatrix} a_{20} & 0 & 0 \\ 0 & -\frac{a_{20}}{2} + 3a_{22} & 3b_{22} \\ 0 & 3b_{22} & -\frac{a_{20}}{2} - 3a_{22} \end{bmatrix} \tag{1}$$

where a_{20}, a_{22}, and b_{22} are the fabric tensor parameters. For cross-anisotropic material, $a_{22} = b_{22} = 0$, and the parameter a_{20} defines the degree of anisotropy.

2.3 Introduction of a Novel Contact Law

A novel contact law is introduced into the CH micromechanical model to simulate granular materials with fabric anisotropy. The normal stiffness and shear stiffness are defined as:

$$k_n = k_{n0}\left(\frac{f_n}{f_{\text{ref}}}\right)^{n_e}, k_r = k_{rR}k_n \tag{2}$$

where k_{n0}, n_e, k_{rR} are model parameters, f_{ref} is a reference force. The yield surface is borrowed from the SANISAND model (Dafalias and Manzari, 2004; Taiebat and Dafalias, 2008):

$$F = (f_r - f_n\alpha)^2 - m^2 f_n^2\left[1 - \left(\frac{f_n}{f_{n0}}\right)^n\right] = 0 \tag{3}$$

where f_n and f_r are the normal and shear contact forces, respectively, α is the rotation hardening variable, m and n are shape parameters, and f_{n0} is the reference normal force. The flow rule is defined as:

$$\dot{u}_n^p = \langle L\rangle\left[Dr_{\text{ef}} + \exp(-Vr_{\text{ef}})\right],\ \dot{u}_r^p = \langle L\rangle\left[sr_{\text{ef}} + X\eta\exp(-Vr_{\text{ef}})\right] \tag{4}$$

where $\langle L\rangle$ is the plastic multiplier, $\dot{u}_n^p$ and $\dot{u}_r^p$ are the normal and shear plastic displacement, respectively, $D = sA_d\left(\alpha^d - \alpha\right)$, $r_{\text{ef}} = |\eta - \alpha|$, X and V are model parameters, and s is the sign of $\eta - \alpha$. The hardening rule for α is defined as:

$$\mathrm{d}\alpha = \langle L\rangle hr_{\text{ef}}\left(\alpha^b - \alpha\right) \tag{5}$$

where h, ρ_c, θ are model parameters and $\alpha^{c,b,d}$ is defined as:

$$\alpha^{c,b,d} = s\alpha_c^{c,b,d}, \alpha_c^c = \tan\phi_c - m, \alpha_c^b = \alpha_c^c\left(\frac{e_c}{e}\right)^{n_b}, \alpha_c^d = \alpha_c^c\left(\frac{e_c}{e}\right)^{n_d} \tag{6}$$

where e and e_c are the current and critical void ratios, respectively, ϕ_c is the critical friction angle, and n_b and n_d are model parameters. The critical state line is defined as (Yin et al., 2018):

$$e_c = e_{ref}\exp\left[-\lambda\left(\frac{p}{p_{ref}}\right)^{\xi}\right] \tag{7}$$

where e_{ref}, λ, and ξ are model parameters, p is the mean effective stress, and p_{ref} is the reference stress.

3 Evaluation of the Proposed Model

3.1 Calibration of Model Parameters

To evaluate the proposed model, drained triaxial experiments were collected from the literature. Table 1 summarizes these experiments.

Table 1. Summary of experiments for model evaluations

Sand	d_{50} (mm)	p_0 (kPa)	Anisotropy	e_0	Reference
Hostun	1.30	100–800	Isotropic	0.502–0.722	Biarez and Hicher (1994)
Cambria	1.42	50–400	Cross-anisotropic	0.535	Ochiai and Lade (1983)

The CMA-ES optimization method (Hansen and Ostermeier, 1996) was employed to determine the model parameters where the objective function was defined as the square root of the sum of squared differences between the experimental and simulated results. The identified parameters are presented in Table 2. Other parameters are the same as those in Taiebat and Dafalias (2008) for Toyoura sand.

Table 2. Calibrated model parameters for experiments (k_{n0}: N/mm, ϕ_c: : degree)

Sand	k_{n0}	n_e	k_{rR}	ϕ_c	e_{ref}	λ	ξ	n_b	n_d	A_d	h	a_{20}
Hostun	417	0.04	0.26	31.3	1.20	0.43	0.15	1.31	2.58	0.66	5.34	-
Cambria	25.4	0.85	4.09	28.7	2.54	0.30	0.48	0.29	1.81	0.33	15.3	0.2

3.2 Evaluation of Model Performance

Figure 2 presents a comparison between the simulations and experimental results for Hostun sand. The experiments were conducted under drained condition with different confining pressures and densities. The results demonstrate that the model simulations align closely with the experimental data in terms of deviatoric stress and void ratio. The model effectively captures the fundamental characteristics of the sand, including stress softening and dilative behavior in dense sand, as well as stress hardening and contractive behavior in loose sand. At the critical state, both the deviatoric stress and void ratio converge to constant values, consistent with the principles of critical state theory.

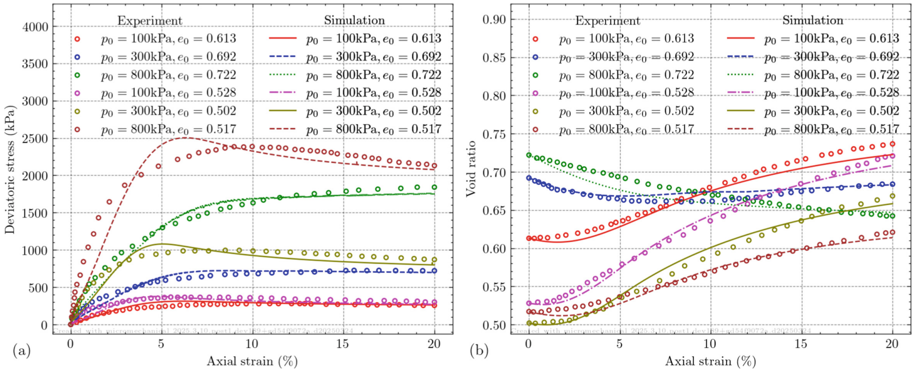

Fig. 2. Simulations for experiments on Hostun sand: (a) deviatoric stress; (b) void ratio

Figure 3 presents a comparison between the simulations and experimental results for Cambria sand. The experiments were conducted under drained condition with different confining pressures and loading directions. The results indicate that the model simulations are in good agreement with the experimental data in terms of deviatoric stress and volumetric strain. The soil specimens appear stiffer when the axial loading is parallel to the direction of deposition (coaxial loading) in terms of deviatoric stress. For the volumetric strain, the soil specimens exhibit a lower compressibility and a higher dilatancy when the axial loading is parallel to the direction of deposition. In general, the model captures the fabric anisotropy of the Cambria sand, which is consistent with the experimental observations.

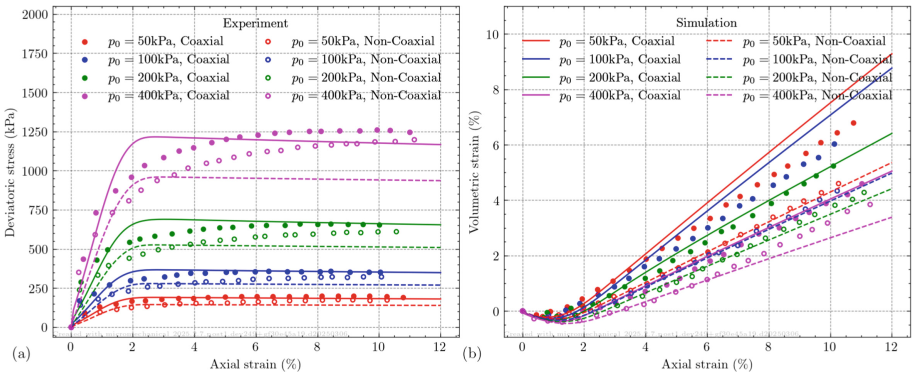

Fig. 3. Simulations for experiments on Cambria sand: (a) deviatoric stress; (b) void ratio

4 Limitations and Future Work

The proposed model is effectively calibrated under triaxial loading conditions, accounting for the fabric anisotropy of granular materials. However, it has not yet been validated under general loading conditions, such as true triaxial, plane strain, or cyclic loading. Future research will focus on validating the model under these conditions. Additionally, the micromechanical model can be implemented as a user-defined material model in finite element software (e.g., Abaqus) to simulate granular material behavior in boundary value problems (Zhao et al., 2018). It should be noted that while the micromechanical model is computationally more efficient than discrete element method (DEM) simulations, it remains more expensive than conventional phenomenological models due to the integration of microscale contact forces. Approximately, the computational cost of the micromechanical model is 37 times higher than that of the SANISAND model when 37 contacts are considered at the microscale.

5 Conclusions

This paper presents a multi-scale micromechanical model that integrates a novel contact law. The proposed model accounts for fabric anisotropy without relying on the fabric tensor used in critical state theory, which is empirical and lacks physical meaning. The model is validated through experiments on Hostun sand and Cambria sand. The results indicate that the model effectively captures the fundamental characteristics of sand, including stress softening and dilative behavior in dense sand, hardening and contractive behavior in loose sand. Moreover, the model captures the fabric anisotropy of the sand, which is consistent with the experimental observations. Overall, the proposed model offers a promising alternative for simulating granular materials with fabric anisotropy in geotechnical applications.

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